EQUIVARIANT RESOLUTION OF POINTS OF INDETERMINACY

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Abstract. We prove an equivariant version of Hironaka's theorem on elimination of points of indeterminacy. Our proof relies on canonical resolution of singularities.

1. Introduction

Throughout this note we shall work over an algebraically closed field of characteristic 0. All algebraic varieties, schemes, groups, and all maps between them will be defined over k. The main objects of interest for us will be algebraic varieties with a G-action; we will refer to them as G-varieties. A G-equivariant morphism between two such varieties will be called a morphism of G-varieties. The terms "rational map of G-varieties", "birational morphism of G-varieties", "birational isomorphism of G-varieties", etc., are defined in a similar manner.

Hironaka's theorem on elimination of points of indeterminacy (see [Hi, §0.5, Question E and Main Theorem II]) asserts that every rational map $f\colon X \dashrightarrow Y$ can be resolved into a regular map by a sequence of blowups $\pi\colon X_m \longrightarrow \dots \longrightarrow X_0 = X$ with smooth centers. In other words, π can be chosen so that the composition $f\pi$ is regular. The purpose of this paper is to prove the following equivariant version of this result.

Theorem 1. *Let $f\colon X \dashrightarrow Y$ be a rational map of G-varieties where Y is complete. Then there is a sequence of blowups*

$$\pi\colon X_m \longrightarrow X_{m-1} \longrightarrow \dots \longrightarrow X_1 \longrightarrow X_0 = X \qquad (1)$$

with smooth G-invariant centers such that the composition $f\pi$ is regular.

Our proof will rely on canonical resolution of singularities. Along the way we prove an equivariant form of Chow's lemma (Proposition 5), generalizing a theorem of Sumihiro ([Su, Theorem 2]).

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2. Birational morphisms as blowups

The following result is an equivariant analogue of [Ha, Theorem 7.17].

Proposition 2. *Let $f\colon X' \longrightarrow X$ be a birational proper morphism of G-varieties, where X is smooth and X' is quasiprojective. Then there exists a G-invariant sheaf of ideals $\mathcal{I}$ on X such that X' is the blowup of $\mathcal{I}$.*

Proof. Let $\sigma\colon G\times X \longrightarrow X$ be the given action of G on X and $\mathrm{pr}_2\colon G\times X \longrightarrow X$ be the projection onto the second factor.

By a theorem of Kambayashi [Ka], there is an action of G on the projective space $\mathbb{P}^n$ (via a representation $G \longrightarrow \mathrm{PGL}_{n+1}$) and a G-equivariant embedding $X' \hookrightarrow \mathbb{P}^n$; this yields a G-equivariant embedding $i\colon X' \hookrightarrow \mathbb{P}^n\times X$.

Here $\mathbb{P}^n \times X$ is a projective space over X; set $\mathcal{L} = i^*\mathcal{O}_{\mathbb{P}^n\times X}(1)$ and $\mathcal{T} = \bigoplus_{d=0}^{\infty} f_*(\mathcal{L}^d)$, where $\mathcal{L}^0 = \mathcal{O}_{\mathbb{P}^n\times X}$. Let $\mathcal{T}_1 = f_*\mathcal{L}$ be the component of $\mathcal{T}$ of degree one. The action of G on $\mathbb{P}^n \times X$ yields a G-linearization of the sheaf $\mathcal{T}_1$, i.e., an isomorphism $\sigma^*\mathcal{T}_1 \cong \mathrm{pr}_2^*\,\mathcal{T}_1$ which satisfies the same cocycle condition as in the definition of G-linearization of an invertible sheaf (see, e.g., [MFK, Definition 1.6]); informally speaking, G acts on the pair $(X, \mathcal{T}_1)$.

We refer to the proof of [Ha, Theorem 7.17] for the following facts:

1. After replacing the embedding i by its e-fold embedding for some positive integer e (thus replacing $\mathcal{L}$ by $\mathcal{L}^e$), we may assume that the graded $\mathcal{O}_X$-algebra $\mathcal{T}$ is generated by $\mathcal{T}_1$.
2. $X' \cong \operatorname{Proj}\mathcal{T}$.
3. Assume $\mathcal{T}$ is generated by $\mathcal{T}_1$ as in (1). If there is an invertible sheaf $\mathcal{M}$ on X and a sheaf of ideals $\mathcal{I}$ on X such that $\mathcal{I} \cong \mathcal{T}_1 \otimes \mathcal{M}$, then X' is isomorphic to the blowup of $\mathcal{I}$.

The variety X is smooth, and hence, for any sheaf of ideals $\mathcal{F}$ on X of rank one without torsion, its dual $\mathcal{F}^* = \mathcal{H}\mathrm{om}(\mathcal{F}, \mathcal{O}_X)$ is an invertible sheaf. (To see this, note that locally at any point $x \in X$, the generator of $\mathcal{F}_x^*$ is given by the homomorphism $\mathcal{F}_x \longrightarrow \mathcal{O}_{X,x}$ which maps the generators of $\mathcal{F}_x$ as an $\mathcal{O}_{X,x}$-module of rank one, into elements of $\mathcal{O}_{X,x}$ not having a nontrivial common multiple; such homomorphism is unique up to an invertible multiple since the local ring $\mathcal{O}_{X,x}$ is regular, and hence, factorial.)

Thus the second dual $\mathcal{T}_1^{**}$ is an invertible sheaf, and we have an embedding $\mathcal{T}_1 \hookrightarrow \mathcal{T}_1^{**}$. The G-linearization of $\mathcal{T}_1$ yields a G-linearization of $\mathcal{T}_1^{**}$, and the above embedding is, in fact, an embedding of G-linearized sheaves. Taking $\mathcal{M} = (\mathcal{T}_1^{**})^{-1}$, we see that X' is isomorphic to the blowup of the sheaf of ideals $\mathcal{I} = \mathcal{T}_1 \otimes (\mathcal{T}_1^{**})^{-1}$.

The G-linearizations of $\mathcal{T}_1$ and $\mathcal{T}_1^{**}$ yield a G-linearization of $\mathcal{I}$ and a G-linearized embedding $\mathcal{I} = \mathcal{T}_1 \otimes (\mathcal{T}_1^{**})^{-1} \hookrightarrow \mathcal{T}_1^{**} \otimes (\mathcal{T}_1^{**})^{-1} = \mathcal{O}_X$. This shows that $\mathcal{I}$ is a G-invariant sheaf of ideals on X. □

3. Canonical simplification of a finite collection of ideals

One of the main resolution theorems of Hironaka [Hi, Main Theorem II] asserts that any finite collection of sheaves of ideals $\{\mathcal{I}_i\}$ can be "simplified"

by a finite sequence $\pi\colon X_m \longrightarrow \ldots \longrightarrow X_0 = X$ of blowups with smooth centers. In other words, the sequence of blowups can be chosen so that $\pi^*\mathcal{I}_i$ is locally principal for each i, and is locally generated by a monomial with respect to a normal crossing divisor.

Bierstone and Milman [BM, Theorem 1.10] proved that any sheaf of ideals $\mathcal{I}$ on an algebraic variety X can be "simplified" in this sense in a canonical way, so that the sequence π is canonically defined; in particular, every automorphism of X preserving $\mathcal{I}$ lifts to the entire sequence; see [BM, Remark 1.5]. This immediately implies that any finite *ordered* collection of sheaves of ideals $\{\mathcal{I}_i\}$ can be "simplified" in a canonical way.

In this section we will show that a finite *unordered* collection of sheaves of ideals can be simplified in a similar manner. This result will be used in the proof of Theorem 1.

Proposition 3. *Let X be an algebraic variety and $\{\mathcal{I}_1, \ldots, \mathcal{I}_n\}$ be a finite collection of sheaves of ideals on X. Then there exists a sequence of blowups*

$$\pi\colon X_m \longrightarrow X_{m-1} \longrightarrow \ldots \longrightarrow X_1 \longrightarrow X_0 = X \tag{2}$$

such that $\pi^*\mathcal{I}_i$ is locally principal for each i and any automorphism of X that preserves (but possibly non-trivially permutes) the collection $\{\mathcal{I}_1, \ldots, \mathcal{I}_n\}$ lifts to the entire sequence (2)*.*

Proof. To motivate our construction, we begin with the following observation. Let $V(\mathcal{I}_i)$ be the subscheme of X cut out by $\mathcal{I}_i$. If the subschemes $V(\mathcal{I}_i)$ are pairwise disjoint, i.e., if $\mathcal{I}_i + \mathcal{I}_j = \mathcal{O}_x$ for any $i \neq j$, then the proposition follows immediately from [BM, Theorem 1.10 together with Remark 1.5]: indeed, a sequence of blowups that simplifies their intersection $\mathcal{I}_1 \cap \cdots \cap \mathcal{I}_n$, will simplify each $\mathcal{I}_i$.

The idea of the proof is to reduce the general case to the case where every i-fold intersection of $V(\mathcal{I}_1), \ldots, V(\mathcal{I}_n)$ is empty (i.e. the sum of any i of the sheaves $\mathcal{I}_1, \ldots, \mathcal{I}_n$ equals $\mathcal{O}_X$) first for $i = n$, then for $i = n-1$, etc., until we reach $i = 2$. We use descending induction on i. For the base case we can take $i = n+1$, where the condition we are interested in is trivially satisfied.

Let $S_\Lambda = \sum_{j \in \Lambda} \mathcal{I}_j$, where Λ is a subset of $\{1, \ldots, n\}$. For the induction step, assume

$$S_\Lambda = \mathcal{O}_X \text{ whenever } |\Lambda| = i, \tag{3}$$

for some $i \geq 2$. Set $\mathcal{J} = \bigcap_{|\Omega| = i-1} S_\Omega$. Note that by our assumption $S_{\Omega_1} + S_{\Omega_2} = \mathcal{O}_X$ for any two distinct subsets Ω_1 and Ω_2 of $\{1, \ldots, n\}$ of cardinality $i-1$, so that $V(\mathcal{J})$ is the disjoint union of $V(S_\Omega) = \bigcap_{j \in \Omega} V(\mathcal{I}_j)$ with $|\Omega| = i-1$.

Let $\pi\colon X' \longrightarrow X$ be the canonical simplification of the sheaf $\mathcal{J}$; as we have seen above, π simplifies each S_Ω with $|\Omega| = i-1$. For $j = 1, \ldots, n$, denote the conductor $(\pi^*\mathcal{I}_j) : (\pi^*\mathcal{J})$ by $\mathcal{I}'_j$; it is natural to think of $\mathcal{I}'_j$ as a "weak transform" of $\mathcal{I}_j$. The stalk of this sheaf of ideals at a (not necessarily closed) point $x \in X'$ is described as follows.

If $x \notin V(\pi^*\mathcal{I}_j)$ then $(\mathcal{I}'_j)_x = (\pi^*\mathcal{I}_j)_x = \mathcal{O}_{x,X'}$. If $x \in V(\pi^*\mathcal{I}_j)$ and $x \in V(\pi^* S_\Omega)$ for some Ω satisfying $|\Omega| = i-1$, then such Ω is unique and $j \in \Omega$ (otherwise $V(\pi^* S_{\Omega\cup\{j\}})$ would be nonempty, contrary to (3)). Thus in this case $\pi^*\mathcal{I}_j \subset \pi^* S_\Omega$ and $(\mathcal{I}'_j)_x = (\pi^* S_\Omega)_x^{-1}(\pi^*\mathcal{I}_j)_x$, where $(\pi^* S_\Omega)_x \subset \mathcal{O}_{x,X'}$ is a principal ideal. Finally, if $x \in V(\pi^*\mathcal{I}_j)$ and $x \notin V(\pi^* S_\Omega)$ for any Ω satisfying $|\Omega| = i-1$, then $x \notin V(\pi^*\mathcal{J})$ and $(\mathcal{I}'_j)_x = (\pi^*\mathcal{I}_j)_x$. To summarize:

$$(\mathcal{I}'_j)_x = \begin{cases} (\pi^* S_\Omega)_x^{-1}(\pi^*\mathcal{I}_j)_x & \text{if } x \in V(\pi^* S_\Omega) \text{ for some } \Omega \subset \{1,\dots,n\} \\ & \text{such that } j \in \Omega \text{ and } |\Omega| = i-1, \\ (\pi^*\mathcal{I}_j)_x & \text{otherwise.} \end{cases} \tag{4}$$

Consequently, for any sequence of blowups $\pi' \colon X'' \longrightarrow X'$, the ideal $(\pi')^*\mathcal{I}'_j$ is locally principal if and only if the ideal $(\pi')^*(\pi^*\mathcal{I}_j)$ is locally principal. This reduces the problem of simplifying the collection $\{\mathcal{I}_1, \cdots, \mathcal{I}_n\}$ of sheaves of ideals on X to the problem of simplifying the collection $\{\mathcal{I}'_1, \cdots, \mathcal{I}'_n\}$ of sheaves of ideals on X'.

For $\Lambda \subset \{1,\dots,n\}$, set $S'_\Lambda = \sum_{j\in\Lambda} \mathcal{I}'_j$. We claim that

$$S'_\Lambda = \mathcal{O}_{X'} \text{ whenever } |\Lambda| = i-1. \tag{5}$$

We will prove this equality by showing that $(S'_\Lambda)_x = \mathcal{O}_{X',x}$ for every $x \in X'$. Indeed, if $x \notin V(\pi^*(\mathcal{I}_j))$ for some $j \in \Lambda$ then

$$\mathcal{O}_{X',x} = (\pi^*\mathcal{I}_j)_x \subset (\mathcal{I}'_j)_x \subset (S'_\Lambda)_x\,,$$

as desired. On the other hand, if $x \in V(\pi^*\mathcal{I}_j)$ for every $j \in \Lambda$, i.e., $x \in V(\pi^* S_\Lambda)$, then (4) tells us that

$$(S'_\Lambda)_x = \sum_{j\in\Lambda}(\mathcal{I}'_j)_x = \sum_{j\in\Lambda}(\pi^* S_\Lambda)_x^{-1}(\pi^*\mathcal{I}_j)_x = (\pi^* S_\Lambda)_x^{-1}(\pi^* S_\Lambda)_x = \mathcal{O}_{X',x}\,.$$

We have thus reduced the problem of simplifying the collection $\{\mathcal{I}_1, \cdots, \mathcal{I}_n\}$ of sheaves of ideals on X, satisfying condition (3), to the problem of simplifying the collection $\{\mathcal{I}'_1, \dots, \mathcal{I}'_n\}$ of sheaves of ideals on X', satisfying condition (5). This completes the induction step.

To finish the proof of the proposition, note that the sequence (2) of blowups constructed by the recursive algorithm we just described, depends on X and the unordered collection $\{\mathcal{I}_i\}$ in a canonical way; see [BM, Remark 1.5]. Hence, any automorphism of X that preserves the unordered collection $\{\mathcal{I}_i\}$, lifts to to the entire sequence (2), as claimed. $\square$

Remark 4. Our proof also shows that each $\pi^*\mathcal{I}_i$ is generated by a monomial with respect to a normal crossing divisor. In other words, π simplifies each $\mathcal{I}_i$ in the sense of Hironaka's original definition; for details see [BM, Remark 1.8]. This assertion will not be used in the sequel; for this reason we did not include it in the statement of Proposition 3.

4. Equivariant Chow Lemma

In this section we will prove the following generalization of Chow's lemma.

Proposition 5. *For every G-variety X, there exists a quasiprojective G-variety Z and a proper birational morphism $Z \longrightarrow X$. If X is complete then Z is projective.*

Note that if G is assumed to be connected, this result is a well-known theorem of Sumihiro [Su, Theorem 2]; see also [PV, Theorem 1.3]. The argument below reduces the general case to the case where G is connected.

Proof. The second assertion is an immediate consequence of the first: if X is complete, $Z \longrightarrow X$ is proper and Z is quasiprojective then Z is also complete and, hence, projective.

To prove the first assertion let G_0 be the connected component of G. Applying Sumihiro's theorem to X, viewed as a G_0-variety, yields a quasiprojective G_0-variety Y and a proper G_0-equivariant birational morphism $f\colon Y \longrightarrow X$.

For the rest of this proof we shall use set-theoretic notation: by a "point" we will always mean a closed point.

Recall that the homogeneous fiber product $G*_{G_0}Y$ is the G-variety defined as the geometric quotient $(G\times Y)/G_0$ for the action of G_0 given by $g_0(g,y) = (gg_0^{-1}, g_0y)$, where $g\in G$, $g_0 \in G_0$ and $y \in Y$; see [PV, Section 4.8]. We shall write $[g,y]$ for the element of $G*_{G_0}Y$ represented by $(g,y)\in G\times Y$. Since G_0 has finite index in G, $G*_{G_0}Y$ admits a more concrete description as a disjoint union of $|G/G_0|$ copies of Y. More precisely, if we choose a representative a_h for each $h \in G/G_0$, we can explicitly identify $G/G_0\times Y$ and $G*_{G_0}Y$ as abstract varieties, via $(h,y)\mapsto [a_h,y]$. Moreover, if we define a G-action on $G/G_0\times Y$ by $g(h,y) \longrightarrow (\overline{g}h, (a_h^{-1}g^{-1}a_{\overline{g}h})y)$ then $(h,y)\mapsto [a_h,y]$ identifies $G/G_0\times Y$ and $G*_{G_0}Y$ as G-varieties. Here $\overline{g}$ is the image of g in G/G_0 and $(a_h^{-1}g^{-1}a_{\overline{g}h})y$ is well-defined because $a_h^{-1}g^{-1}a_{\overline{g}h}$ is an element of G_0.

Let $\alpha\colon G*_{G_0}Y \longrightarrow X$ and $\beta\colon G*_{G_0}Y \longrightarrow G/G_0$ be the maps of G-varieties given by $\alpha\colon [g,y] \longrightarrow gf(y)$ and $\beta\colon [g,y]\mapsto \overline{g}$. (Here G acts on G/G_0 by left multiplication.) These maps are shown in the diagram below.

$$\begin{tikzcd} & G*_{G_0}Y \simeq G/G_0\times Y \arrow[dl, "\alpha"'] \arrow[dr, shift left, "\beta"] & \\ X & & G/G_0 \arrow[ul, shift left, "s"] \end{tikzcd}$$

Let S be the set of all sections s of β. Note that if we identify $G*_{G_0}Y$ with $G/G_0\times Y$ as above, then $\beta\colon G/G_0\times Y \longrightarrow G/G_0$ is the projection to the first factor. Thus $S \simeq Y^{|G/G_0|}$ as an abstract variety. Moreover, since β is G-equivariant, G acts on this variety by $g\colon s\mapsto t$, where $s,t\in S$ and $t(h) = g\cdot s(\overline{g}^{-1}h)$ for any $h\in G/G_0$.

Let Z be the closed G-invariant subvariety of S consisting of those sections $s\colon G/G_0 \longrightarrow G*_{G_0}Y \simeq G/G_0\times Y$ with the property that $\alpha\circ s(G/G_0)$ is a single point of X; we shall denote this point by x_s.

We claim that the morphism $\phi\colon Z \longrightarrow X$ given by $s \longrightarrow x_s$ has the properties asserted in the proposition. Indeed, since Y is quasiprojective, and Z is a closed subvariety of $S \simeq Y^{|G/G_0|}$, Z is quasiprojective as well.

To show that ϕ is a birational morphism, assume the birational morphism $f\colon Y \longrightarrow X$ is an isomorphism over a dense open subset $U \subset X$. Then $V = \bigcap_{h \in G/G_0} a_h U$ is also a dense open subset of X, and for every $x \in V$, $\alpha^{-1}(x) = \{[a_h, f^{-1}(a_{h^{-1}}x)] \,:\, h \in G/G_0\}$; it is the image of the unique section $s_x \in S$ satisfying $x_{s_x} = x$. This section is given by $s_x(h) = [a_h, f^{-1}(a_h^{-1}x)]$, and the morphism $V \longrightarrow Z$, $x \mapsto s_x$, is a two-sided rational inverse to ϕ. $\square$

5. Proof of Theorem 1

We begin with two reductions. First of all, we may assume without loss of generality that Y is projective. Indeed, Proposition 5 yields a projective G-variety Y' and a G-equivariant birational morphism $u\colon Y' \longrightarrow Y$. We can now replace Y by Y' and f by $f' = u^{-1}f\colon X \dashrightarrow Y'$ If we can construct a sequence of blowups π, as in (1), so that $f'\pi$ is regular, then $f\pi$ is regular as well, i.e., the same sequence of blow ups will resolve the indeterminacy locus of f.

Secondly, we may assume that X is smooth. Indeed, let

$$X_l \xrightarrow{\pi_l} \dots \xrightarrow{\pi_1} X_0 = X \ , \tag{6}$$

be the canonical resolution of singularities of X, as in [V, Theorem 7.6.1] or [BM, Theorem 13.2]. Here X_l is smooth, the centers $C_i \subset X_i$ are smooth and G-invariant, and the action of G lifts to the entire resolution sequence (6). Replacing X by X_l, we may assume that X is smooth.

From now on we will assume X is smooth and Y is projective. Let G_0 be the connected component of G. As X is smooth, it is normal, and we can apply a theorem of Sumihiro (see [Su, Theorem 1] or [KKLV, Theorem 1.1] or [PV, Theorem 1.2]) which yields a finite covering $X = U_1 \cup \dots \cup U_d$, where each U_i is a G_0-invariant quasiprojective open subvariety of X.

For each i, let Z_i be the closure of the graph of $f|_{U_i}\colon U_i \dashrightarrow Y$ in $U_i \times Y$; it is a quasiprojective variety, and the projection $h_i\colon Z_i \longrightarrow U_i$ is a proper birational morphism of G-varieties. By Proposition 2, we can find a G_0-invariant sheaf of ideals $\mathcal{I}_i$ on U_i such that Z_i is isomorphic to the blowup of $\mathcal{I}_i$. Let $\mathcal{I}_i'$ be the maximal sheaf of ideals on X such that $\mathcal{I}_i'|_{U_i} = \mathcal{I}_i$; as $\mathcal{I}_i'$ is unique, it is G_0-invariant. The G-invariant collection of sheaves $\{g^*\mathcal{I}_i' \,|\, g \in G,\ i = 1, \dots, d\}$ on X is finite (it contains no more than $|G/G_0|$ sheaves for each i). Proposition 3 yields a G-equivariant sequence of blowups

$$\pi\colon X_m \longrightarrow X_{m-1} \longrightarrow \dots \longrightarrow X_1 \longrightarrow X_0 = X$$

with the property that the pullback $\pi^*\mathcal{I}_i'$ is locally principal for each i; hence, the composition $h_i^{-1}\pi\colon X_m \longrightarrow X \dashrightarrow Z_i$ is regular on $\pi^{-1}(U_i)$. This implies that $f\pi\colon X_m \longrightarrow X \dashrightarrow Z_i \longrightarrow Y$ is regular on $\pi^{-1}(U_i)$ for each i, and hence, on all of X_m. $\square$

Remark 6. Note that if Y is assumed to be projective in the statement of Theorem 1 then Proposition 5 is not needed in the proof. On the other hand, if G is assumed to be connected, then Proposition 3 may be replaced by [BM, Theorem 1.10 together with Remark 1.5] and Proposition 5 may be replaced by [Su, Theorem 2].

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